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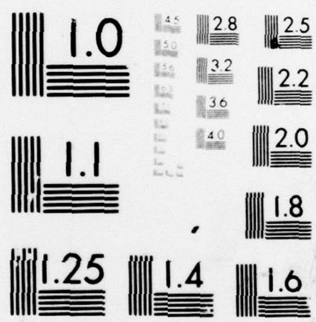

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Using majorization and Schur-functions, Marshall, Olkin, and Proschan obtained a result concerning monotonicity of the ratio of means. This note shows that a slight extension of their result provides a unified method for obtaining and extending inequalities between means due to Chan, Goldberg, and Gonek, as well as deriving additional inequalities of the same type.

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by

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ABSTRACT

Using majorization and Schur-functions, Marshall, Olkin, and Proschan obtained a result concerning monotonicity of the ratio of means. This note shows that a slight extension of their result provides a unified method for obtaining and extending inequalities between means due to Chan, Goldberg, and Gonek, as well as deriving additional inequalities of the same type.

1. Introduction. Chan, Goldberg, and Gonek [1] show that:

$$\text{(1)} \qquad \left[\frac{x^p + y^p}{(1-x)^p + (1-y)^p}\right]^{1/p} < \left[\frac{x^q + y^q}{(1-x)^q + (1-y)^q}\right]^{1/q},$$

where $0 \le x < y$, $x + y < 1$, and $p < q$; and

$$\text{(2)} \qquad \left[\frac{\sum_{i=1}^{n} x_i^{-p}}{\sum_{i=1}^{n} (1-x_i)^{-p}}\right]^{-1/p} \le \left[\frac{\sum_{i=1}^{n} x_i^{-q}}{\sum_{i=1}^{n} (1-x_i)^{-q}}\right]^{-1/q}$$

where $0 \le x_i \le 1/2$ and $p > 0$. Strict inequality holds in (2) unless $x_1 = x_2 = \ldots = x_n$.

Earlier, Marshall, Olkin, and Proschan [2] showed:

$$\text{(3)} \qquad \left[\frac{\sum_{i=1}^{n} \lambda_i a_i^r}{\sum_{i=1}^{n} \lambda_i b_i^r}\right]^{1/r} \text{ is increasing in } r,$$

where $a_1 \ge a_2 \ge \ldots \ge a_n > 0$, $b_1 \ge b_2 \ge \ldots b_n > 0$, $\frac{b_1}{a_1} \le \frac{b_2}{a_2} \le \ldots \le \frac{b_n}{a_n}$, and $\lambda_i > 0$, $i = 1, \ldots, n$, $\sum_{i=1}^{n} \lambda_i = 1$.

Result (3) was obtained using majorization and Schur-functions (for definitions see [2]).

The main purposes of this note are to show that using (3), (a) inequalities (1) and (2) can be proved in a *unified* way, (b) (1) and (2) can be *extended*, and (c) *additional* inequalities of a similar type can be obtained.

2. Main Results. Before we state and prove the main results, we present several remarks:

Remark 2.1. It is easy to verify that (3) holds even if certain of the a_i's are equal to zero.

Remark 2.2. Careful inspection of the proof of (3) shows that in certain cases the ratio in (3) is *strictly* increasing in r.

We may now prove:

Theorem 2.3. Let $0 \le x < y$, $x + y < 1$, $0 < \lambda < 1$, and $p < q$. Then

$$(4) \quad \left[\frac{\lambda x^p + (1-\lambda)y^p}{\lambda(1-x)^p + (1-\lambda)(1-y)^p}\right]^{1/p} < \left[\frac{\lambda x^p + (1-\lambda)y^q}{\lambda(1-x)^q + (1-\lambda)(1-y)^q}\right]^{1/q}.$$

Proof. Clearly $(1 - x)x < (1 - y)y$. Let $a_1 \equiv y$, $a_2 \equiv x$, $b_1 \equiv 1 - x$, and $b_2 \equiv 1 - y$. Inequality (4) follows from (3) by Remark 2.2. ||

Setting $\lambda = \frac{1}{2}$ in (4) we get (1) as a special case.

The same technique yields an extension of Inequality (2):

Theorem 2.4. Let $0 \le x_i \le \frac{1}{2}$, $i = 1, \ldots, n$, $p > 0$, $\lambda_i \ge 0$, $i = 1, \ldots, n$, and $\sum_1^n \lambda_i = 1$. Then

$$(5) \quad \left[\frac{\sum_{i=1}^n \lambda_i x_i^{-p}}{\sum_{i=1}^n \lambda_i (1 - x_i)^{-p}}\right]^{-1/p} < \left[\frac{\sum_{i=1}^n \lambda_i x_i^{p}}{\sum_{i=1}^n \lambda_i (1 - x_i)^{p}}\right]^{1/p}$$

unless $x_1 = x_2 = \ldots = x_n$.

<u>Proof</u>. Let $x_{[1]} \geq x_{[2]} \geq \dots \geq x_{[n]}$ denote the decreasing rearrangement of $x_1, \dots, x_n$ from now on. Let $a_i \equiv x_{[i]}$, $b_i \equiv (1 - x_{[i]})^{-1}$, $i = 1, \dots, n$. Since $-p < p$ and $(1 - x_{[i]})x_{[j]}^{-1} \leq (1 - x_{[j]})^{-1} x_{[i]}$ for $i < j$, we have by (3):

$$\left[\frac{\sum_{i=1}^{n} \lambda_i x_i^{-p}}{\sum_{i=1}^{n} \lambda_i (1 - x_i)^p}\right]^{-1/p} < \left[\frac{\sum_{i=1}^{n} \lambda_i x_i^{p}}{\sum_{i=1}^{n} \lambda_i (1 - x_i)^{-p}}\right]^{1/p} \tag{6}$$

unless $x_1 = x_2 = \dots = x_n$ (see Remark 2.2). The desired result follows from (6). ||

Note that (2) is a special case of (5) by setting $\lambda_i = \frac{1}{n}$, $i = 1, \dots, n$.

Finally, Theorem 2.5 below yields an inequality similar to (1) and (2). This illustrates that majorization and Schur-functions can be used to generate through (3) a host of inequalities similar to (1) and (2).

<u>Theorem 2.5</u>. Let $x_i \geq 0$, $\lambda_i > 0$, $i = 1, \dots, n$, $\sum_1^n \lambda_i = 1$, and $p < q$. Then:

$$\left[\frac{\sum_{i=1}^{n} \lambda_i x_i^{p}}{\sum_{i=1}^{n} \lambda_i (1 + x_i)^p}\right]^{1/p} \leq \left[\frac{\sum_{i=1}^{n} \lambda_i x_i^{q}}{\sum_{i=1}^{n} \lambda_i (1 - x_i)^{q}}\right]^{1/q}. \tag{7}$$

Strict inequality holds in (7) unless $x_1 = x_2 = \dots = x_n$.

<u>Proof</u>. Let $a_i \equiv x_{[i]}$ and $b_i \equiv 1 + x_{[i]}$, $i = 1, \dots, n$. Since $\frac{1 + x}{x}$ is decreasing, we apply (3) to get the desired result. By Remark 2.2, strict inequality holds in (7) unless $x_1 = x_2 = \dots = x_n$. ||

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1. F. Chan, D. Goldberg, and S. Gonek, On extensions of an inequality among means. Proc. Amer. Math. Soc. 42 (1974), 202-207.

2. A. W. Marshall, I. Olkin, and F. Proschan, Monotonicity of ratios of means and other applications of majorization. Inequalities, ed. by O. Shisha. Academic Press, New York (1967), 177-190.

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